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A Theoretical Exposition of General Equilibrium Analysis

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Abstract
The general equilibrium theory believes that any economy is fundamentally characterized with interdependence among markets of commodities. Factor prices depend on the demand and supply of the various inputs. Where consumers' demand for various goods and services depend on their taste and incomes, consumers' incomes depend on the amounts of resources they own and factor prices. The upshot of this discussion is therefore that, there is general interdependence of all variables in the economy: for everything dependents on every other thing. General equilibrium theory allows the tremendous complexity of the real world by viewing the economy as a vast system of mutually interdependent markets. The general equilibrium model has a solution under specific assumptions and observing the assumptions yield an optimal allocation of resources and welfare improvement. The study concludes that the proof of the existence of general equilibrium for a perfect competitive economy is very important as it results in an efficient allocation of resources and improves the welfare in general.
Keywords: General, Equilibrium, Competitive, Interdependence

1. Introduction

In our undergraduate days, we said that microeconomics is all about the economic behaviour of individual decision-making units like consumers, resource owners, and firms. By then, we interpreted this statement to mean that microeconomics views the behaviour of such individual or household and their workings in isolation, each unit or market being considered separately. Such interpretation may not be quite correct. Microeconomics is also concerned in an important way with how these units and these markets fit together. Indeed, some of the intellectually most exiting, and practically most significant, parts of microeconomics deal with the interrelations among individual units and among various markets.

General equilibrium theory, a major aspect of microeconomics, is indeed a unique breakthrough of economic field in the last century (Tian, 2011; 2018). It considers equilibrium in many markets simultaneously, putting into consideration basic structure of an economy: the interdependence of its constituents (Baumol, 1976). An increase in the price of automobiles can lead to reduction in the demand for tires and increase the demand for Bus Rapid Transportation (BRT). An increase in wages may increase the imports, reduces exports and lead to increase in the use of labour saving machinery. Interdependence among markets may lead to a condition that is not applicable in a partial equilibrium structure. The simple equality of demand and supply is not merely a mathematical equality but according to Starr (1997), a stationary point of dynamic process involving the price and quantity adjustments in the market. This of course is the partial equilibrium: the adjustment of price so that demand equals supply in a particular market. The Marshallian partial equilibrium (ceteris paribus) assumption has been criticised over the years for its lack of realism.

General equilibrium theory is inseparably associated with Walras, hence the Walrasian theory of market from Leon Walras (1874). Walras (1874) publication of the elements of pure economics led him to be called the father of the general equilibrium theory. The issue Walras tried to solve was one presented by Cournot. Cournot had argued that, even though it could be displayed that prices would equate supply and demand at equilibrium, it was unsure that equilibrium occurred for all markets simultaneously. To solve this issue, Walras presented an informal argument for the existence of an equilibrium based on the assumption that equilibrium exists whenever the number of equations equals the number of unknowns. He came up with the Walras' Law which states that, considering any particular market, if all other markets in an economy are in equilibrium, then that specific market must also be in equilibrium. This Law is hinged on the mathematical notion that excess market demands (or excess market supplies) must sum to zero. This means that, in an economy with n markets, it is only adequate to solve n-1 simultaneous equations for the attainment of market equilibrium (Cournot, 1838).

However, more rigorous analysis of the existence of equilibria for a competitive economy comprising a finite numbers of economic agents and commodities is chiefly the result of the works of McKenzie (1959); Nikaido (1957); Arrow and Debreu (1954); Klein (1973). More recently general equilibrium theorists have developed empirical general equilibrium model of real economies paving way for the empirical application of mathematical general equilibrium to the theory of distribution, public finance, and international trade, regional and sectorial economies and so on (Ekanem & Iyoha, 2000; Bewley, 2007).

Following this introduction is a simple model of general equilibrium model. The equations and structures of general equilibrium were presented in the next section. Prove of the existence of general equilibrium within competitive and pure exchange models followed conclusion and implications.

2. A Simple Model of General Equilibrium Analysis

Economists develop a general equilibrium model through mathematical approaches with the equilibrium equation system to maximize consumer benefits and enterprise profits (Arrow & Debreu 1954; Quirk & Saposnik 1968; Trinh 2019; Truong & Toan, 2020). The markets of all commodities and all economic factors are interdependent, and the prices in all markets are simultaneously determined. For instance, consumers' demand for various goods and services interrelates with their tastes and incomes. In turn, consumers' incomes are also related with the amounts of resources they own and factor prices. The demand for factors by firms is highly related with the both the state of technical innovation and the demands for the final goods they produce. The demand for these goods depends on consumers' income, which as can be observed in figure 1, depend on the demand for the factors of production. This circular interrelationship in the economic activity within an economy can be represented in a-two sector model: households and firms:

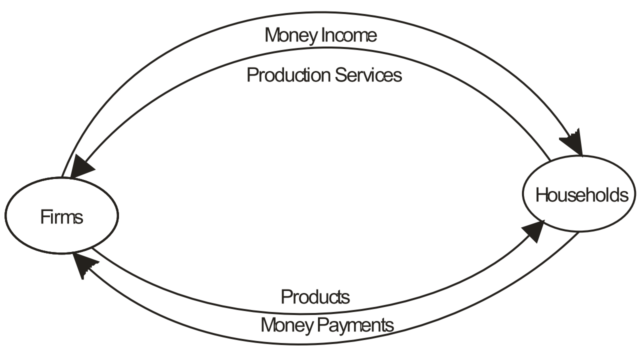

Figure 1: Flows in a Two-Sector Economy

These interrelationships among markets are less visible by the partial equilibrium approach. The interdependence between individuals and market requires that equilibrium for all products and factor markets as well as for all participants in each market must be determined simultaneously so as to attain a consistent set of prices. General equilibrium emerges from the solution of a simultaneous equation model of millions of equations in millions of unknowns. The unknowns are the prices of all factors and all commodities and the quantities purchased and sold by each consumer and each producer.

Suppose an economy has 2,053 different commodities, including bonds, stocks, and factories as well as consumer goods. Let us treat money as another one of those goods, making the total good to be 2,054 in the list. In accord with the discussion of the last section, if hand-sets are item no. 12, the demand for hand-sets will be given by an expression

$$Q_{12} = D_{12}(P_1, P_2, \ldots, P_{2054}, A, M) \quad 1$$

Equation 1 states that the number of hand-set demanded is a function of the price of every one of the 2,054 commodities, P_1, P_2... P_{2054}. Again, demand will depend on the wealth of the nation (presumably a positive correlation between the wealth of the economy and the demands for commodities). The wealth of the economy is a total sum of the variables A (an index of its holdings of physical assets) and M, the amount of money in the circulation. It should be noted that there is no explicit income variable included in this discussion. This is because consumers' income is assumed given by the prices of the commodities which they sell for a living. For example, the price of labour time (the wage rate)-and these prices already appear in the demand function.

There is a similar supply function for item no. 12 hand-sets which may be expressed as

$$S_{12}(P_1, P_2, \ldots, P_{2054}, A, M) \quad 2$$

The economy is agreed to be in a state of general equilibrium if the supply of every commodity is equal to the demand for it. That is, for the 2,054 items in the system, the 2,054 equations must be satisfied:

$$S_1(P_1, P_2, \ldots, P_{2054}, A, M) = D_1(P_1, P_2, \ldots, P_{2054}, A, M) \quad 3$$

$$S_2(P_1, P_2, \ldots, P_{2054}, A, M) = D_2(P_1, P_2, \ldots, P_{2054}, A, M) \quad 4$$

....

$$S_{2054}(P_1, P_2, \ldots, P_{2054}, A, M) = D_{2054}(P_1, P_2, \ldots, P_{2054}, A, M) \quad 5$$

Assuming the values of A and M are given, it then implies several unknown prices as equations and the system can therefore, presumably, be solved for the market clearing values of the prices, P_1, P_2, ..., P_{2054}. Substituting these values into the demand (or supply) expressions will determine the quantities of the various commodities of exchange. This, in essence, is the general equilibrium system and the method by which it determines the prices and quantities sold of the various commodities. It can be expanded and become complicated in various ways, by explicitly including various exogenous variables, that is, variables whose values are determined by non-economic phenomena such as temperature), and endogenous variables (such as advertising expenditure), both of which clearly affect demand. We can also go beyond the supply relationships to explicitly take into cognizance the behaviours of firms and the endowment of natural resources.

3. The Structure of General Equilibrium Model

3.1 Economic Environments

The fundamentals of the economy are economic environments that are exogenously given and characterized by the following terms:

n: the number of consumers (No of buyers)

N = (1; : : : ; n) the set of consumers

J: the number of producers (firms) (market structure)

L: the number of (private) goods (system of economy)

$e = \left((X_i; \geq_i; w_i); (Y_j)\right)$: an economy, or called an economic environment.

$X = X_1 \times X_2 \times :::\times X_n$: consumption space.

$Y = Y_1 \times Y_2 \times :::\times Y_J$: production space.

3.2 Institutional Arrangement: Private Market Mechanism

$p = \left(p^1; p^2; ::::; p^L\right) \in R^L +$: a price vector;

px_i : the expenditure of consumer *i* for *i* = 1; : : : ; *n*;

py_i : the profit of firm *j* for *j* = 1; : : : ; *J*;

pw_i : the value of endowments of consumer *i* for *i* = 1; : : : ; *n*;

$\theta_{ij} \in R_+$: the profit share of consumer *i* from firm *j*, which specifies ownership (property rights) structures, so that

$\sum_{i=1}^{n} \theta ij$ = 1 for j = 1, 2, ..., J and i = 1,...n;

$\sum_{j=1}^{n} \theta ijpyj$ = the total profit dividend received by consumer *i* from firms for *i* = 1; : : : ; *n*.

For *i* = 1; 2; : : : ; *n*, consumer *i*'s budget constraint is given by

$$px_i \leq pw_i + \sum_{j=i}^{j} \theta_{ij} py_j \qquad 6$$

and the budget set is given by

$$B_i\left(p\right) = (x_i \in X_i : px_i \leq pw_i + \sum_{j=i}^{j} \theta_{ij} py_j) \qquad 7$$

A private ownership economy then is referred to

$$e = (e_1, e_2, ..., e_n, \left\{Y_j\right\}_{j=1}^{j}, \left\{\theta_{ij}\right\}) \qquad 8$$

The set of all such private ownership economies are denoted by e.

3.3 Individual Behaviour Assumptions:

(i) Perfect competitive markets: Every player is a price-taker.

(ii) Utility maximization: Every consumer maximizes his preferences subject to $Bi(P)$. That is,

$$\max u_i\left(x_i\right) \qquad 9$$

s.t

$$Px_i \leq Pw_i + \sum_{j=i}^{j} \theta ijPyj \qquad 10$$

(ii) Profit maximization: Every firm maximizes its profit in Yj . That is,

$$\max py_j$$

$$y_j \in Y_j \qquad 11$$

for j = 1, ..., J

3.4 Allocative Competitive Equilibrium

An *allocation* $(x; y)$ is a specification of consumption vector $x = (x1; : : : ; xn)$ and production vector $y = (y1; : : : ; yJ)$.

An allocation $(x; y)$ is individually feasible if $xi \in Xi$ for all $i \in N$, $yj \in Yj$ for all $j = 1; : : : ; J$.

An allocation is weakly balanced

$$\hat{x} \leq \hat{y} + \hat{w} \qquad 12$$

Or specifically

$$\sum_{i=1}^{n} xi \leq \sum_{j=1}^{J} yi + \sum_{i=1}^{n} wi$$

When inequality holds with equality, the allocation is called balanced or attainable. An allocation $(x; y)$ is feasible if it is both individually feasible and (weakly) balanced. Thus, an economic allocation is feasible if the total amount of each good consumed does not exceed the total amount available from both the initial endowment and production.

Denote by $A = \left\{\left(x; y\right) \in X \times Y : \hat{x} \leq \hat{y} + \hat{w}\right\}$ the set of all feasible allocations.

Aggregation:

$\hat{x} = \sum_{i=1}^{n} xi$: aggregation of consumption

$\hat{y} = \sum_{j=1}^{J} yj$: aggregation of production

$\hat{w} = \sum_{i=1}^{n} wi$ aggregation of endowment

4. The Existence of Competitive Equilibrium

The existence of competitive equilibrium can be explained for three cases: (i) the single-valued aggregate excess demand function; (ii) the aggregate excess demand correspondence; (iii) a general class of private ownership production economies. This study is limited to the single-valued aggregate excess demand function which is based on excess demand instead of underlying preference orderings and consumption and production sets. There are many ways to prove the existence of general equilibrium. For instance, one can use:

i. *the Brouwer fixed point theorem approach,*
ii. *KKM lemma approach, and*
iii. *abstract economy approach*

The case we are considering in this study to explain the existence of a competitive equilibrium is the one when the aggregate excess demand correspondence is a single-valued function. A very important property of excess demand function $\hat{z}(p)$ is Walras which can take one of the following three forms:

i. the strong form of Walras' law given by

$P\hat{z}(p) = 0$ for all $P \in R^{L}_{+;}$

ii. the weak form of Walras' law given by

P $\hat{z}(p) \leq 0$ for all $P \in R^{L}_{+;}$;

iii. the interior form of Walras' law given by

P $\hat{z}(p) = 0$ for all $P \in R^{L}_{++;}$

Another important property of excess demand function is homogeneity of $\hat{z}(p)$: it is Homogeneous of degree 0 in price $\hat{z}(\lambda p) = \hat{z}(p)$ for any, λ > 0. From this property, we can normalize prices. Because of homogeneity, for example, we can normalize a price vector as follows:

i. $p'^{1} = \frac{p^{1}}{p^{1}} I = 1, 2, ..., L$

ii. $p'^{1} = p^{1} / \sum_{i=1}^{L} pl$

Thus, without loss of generality, we can restrict our attention to the unit simplex:

$$S^{L-1} = \{ P \in R^{L}_{+}; \sum_{l=1}^{L} pl = 1 \} \qquad 13$$

Then, we have the following theorem on the existence of competitive equilibrium. From the above theorems, Walras' Law is important to prove the existence of a competitive equilibrium. Under which conditions, is Walras' Law held? When each consumer's budget constraint holds with equality:

$$px_i\left(p\right)pw_i+\sum_{j=i}^{j}\theta_{ij}p_{yj} \qquad 14$$

for all *i*, we have

$$\sum_{i=1}^{n}pxi(p)\ =\sum_{i=1}^{n}pwi+\sum_{i=1}^{n}\ \sum_{j=1}^{J}\theta ijpyj(p) \qquad =\sum_{i=1}^{n}Pwi+\sum_{j=1}^{J}pyj(p)$$

which implies that

$$\text{P}[\sum_{i=1}^{n}xi(p)-\sum_{i=1}^{n}wi-\sum_{j=1}^{J}yi(p)=0 \qquad 15$$

so that

$$\text{P. } \hat{z}(p)=0 \qquad (Walras'\ Law) \qquad 16$$

Thus, as long as the budget line holds with equality, Walras' Law must hold. The above existence theorems on competitive equilibrium are based on the assumptions that the aggregate excess demand correspondence is single-valued and satisfies the Walras's Law.

The mathematical proofs of the existence of general equilibrium solutions in the economy rest on a set of mathematical structures called Fixed Point Theorems. The problem of multidimensionality, which characterises general equilibrium analysis, places severe limitations on the graphical illustrations of the existence of general equilibrium. Consequently, mathematical tools have to be explored to explain existence problem. Economists over the years have come to realize that the mathematical proofs to the existence of general equilibrium solutions in the economy rest on mathematical structures called Fixed Point Theorems. The mathematical fixed point theorem maintains essentially that the continuous mapping of a closed, bounded, convex set in an n-Euclidean space into itself leaves at least one point unchanged. This is unknown as the fixed point to which the name is associated.

4.1 Elements of the Brouwer Fixed Point Theorem

We begin by defining a few terms that constitute the major elements in the Brouwer Fixed Point Theorem.

i. Point to point mapping: A point to point mapping in an n-dimensional space is a rule or set of rules which associate a point in (n-dimensional) space with some other point in the space. In general, a mapping in n-dimensional space may be given as

$$X^1 = f^1(x_1, x_2, ..., x_n), i = 1, \qquad 17$$

Or $x = F(x)$ (compact form)

Where x = (x_1, x_2, ...,,xn) and

$$X_i = (x_1^1, x_2^1, ..., x_n^1)$$

The point x^1 is called the image of point x.

ii. A point set in n dimensional space is bounded from above if there exists a set of n finite numbers

$x^* = (x^*1, x^*2, ..., x^*_n)$ $|x^*i \geq xi \quad \forall x = (xi,...,xn)$

iii. A point set in n-dimensional space is bounded from below if there exist a set of n finite number

$x^0 = (x^0{}_1, x^0{}_2, ...,x^0{}_n) \mid x^0{}_i \leq x_i \quad \forall x = (x_1, x_2, ...,x_n)$

iv. A bounded set is bounded both from above and below.

v. Convex sets have already been defined

vi. A point set is closed if and whenever every point of a converging infinite sequence is in the set, the limit point of that sequence is also in the set.

Employing the above definitions as mathematical restrictions, the Brouwer Fixed Point Theorem states that a continuous mapping of a closed and bounded convex set in an n-Euclidean space has a fixed point. That is, if F(x) is mapping, then there exists a point x* in the set which the mapping is defined, such that x* = F(x*).

The Brouwer Fixed Point Theorem is based among other things on the assumption of strict convexity and continuity of consumer utility functions and production functions. The functions are assumed to possess continuous second order partial derivatives that conform to some required restrictions. Thus all system that satisfy the restrictions according to Brouwer proposition possess equilibrium points. But there are systems that possess equilibrium that do not satisfy them.

Multi-dimensionality and other assumptions affecting consumer and producer optimising conditions may lead us to embrace the property of correspondences rather than strict continuity. The property of correspondence that is analogous to continuity of functions is the upper semi-continuity. This is defined as follows:

Let u, v be two topological spaces

Let $c(v) \subset v / c(v) \neq 0$

A correspondence f: u $\rightarrow$(v) is said to be upper semi-continuous if for any $x^0 \in$ u, and any open set w$\in$ v, so that $f(x_0) \subset$ w. Then there exists a neighbourhood N of $x^0/x \in$ N implies that $f(x^0) \subset$ w.

Mapping the point $x^0 \rightarrow$ to the set $f(x^0) \subset$ w in the v space, the function f is upper semi-continuous. This is because all points of the neighbourhood N of x0 are mapped into closed sets contained in the set w. The conclusion is that the images of the points N and x^0 are contained in w.

4.2 The Kakutani Fixed Point Theorem

Based on the above definition of upper semi-continuity, the Kakutani Theorem can be given as follows:

Let K = a closed, bounded convex set in n-Euclidean space

Let ϕ (K) = a family of non-empty, closed convex subsets of K.

If the mapping of f: $K \rightarrow \phi$ (K) is upper semi-continuous then there exists a point $x \in K / x \in f(x)$ and the Kakutani Theorem is satisfied.

In summary, the Brouwer Fixed Point Theorem maintains that under proper conditions, a function maps a point into itself. The Kakutani theorem maintains that under proper conditions, a correspondence maps a point into a set of points containing itself.

4.3 Proof of Existence of Equilibrium Based on a Pure Exchange Model

Analysis of the existence problem began in the early 1930s when Stackelberg (1933), Zeuthen (1933); Schesinger (1935) discovered some of its basic features and Wald (1935, 1936a, 1936b) obtained its first solution. After a discontinuity of about two decades, the issue of existence was taken up again by Arrow and Debreu (1954); McKenzie (1954, 1955); Shoven & Whalley (1995) and many others. The first proof of existence of a competitive equilibrium was obtained by Neumann (1937) and it turned out to be of greater importance of the subject. Neumann proved a topological lemma which, in its reformulation by Kakutani (1941) as a fixed-point theorem for a correspondence and it became the most powerful tool for the proof of existence of a general equilibrium, where equilibrium prices are endogenously determined.

We will begin with the application of the Gale-Nikaido approach in a pure exchange model. Equilibrium in a pure exchange model is given by a set of prices.

$$P^* / \varepsilon_i\left(P^*\right) - w_i \leq 0 \forall_i \qquad 18$$

And $\varepsilon_i\left(P^*\right) - w_i = 0$

Let the excess demand function is defined as:

$$z_i\left(P\right) = \varepsilon_i\left(P\right) - w_i \qquad 19$$

The Gale-Nikaido mapping is modelled by the equation

$$T_i(p) = \frac{p_i + Max\left[0, z_i(p)\right]}{1 + \sum_{i=0}^{N} Max\left[0, z_i(p)\right]} \qquad 20$$

T_i (p) represents the price adjustment process for the i^{th} price.

i. The *T* function is a price adjustment function representing how the (Walrasian) auctioneer manages the price.

ii. The expression pi + zi (p) gives us the notion that price of goods in excess demand

should be raised and those in excess supply should be reduced.

iii. The operator max [0, ∗] represents the idea that adjusted prices should be nonnegative.

vi. The fractionalization of the *T* function represents the normalization of the function that is the proportional re-adjustment of the adjusted prices to fit into the unit simplex.

The denominator is non-zero.

Z(∗) is a continuous function.

Given the above classifications, the operations of max [∗], sum, and division by a non-zero continuous function must be continuous. Thus the equation (20) provides a continuous mapping of the unit simplex into itself. Thus by Brouwers theorem there must exist a fixed point, i.e. a price vector P* such that

$$T_i\left(p^*{}_i\right) = \frac{p_i + Max\left[0, z_i\left(p^*\right)\right]}{1 + \sum_{i=0}^{N} Max\left[0, z_i\left(p^*\right)\right]} \qquad 21$$

Recall that the T function is the auctioneer's price adjustment function. This means that P* is the price at which the auctioneer stops adjusting. The price adjustment rule is that once equilibrium price is reached, the adjustment stops. The proof ends by showing that P* is not just the stopping point of the price adjustment process, but it actually represents the vector of general equilibrium prices for the economy.

In accordance with Starr (1997) and Shoven & Whalley (1995), we do this by invoking Walras' law and ensuring that it applies at this set of prices. This implies that at P* all markets are cleared except, possibly for free goods. Since *T* (P*) = P* then for each goods (i)

Ti(P*) = P*i ∀i = 1,........,N

and

$$p^*_i = \frac{p_i + Max\left[0, z_i\left(p^*\right)\right]}{1+\sum_{i=0}^{N} Max\left[0, z_i\left(p^*\right)\right]} \quad 22$$

To show that P* corresponds to equilibrium price vector for the above model, we define c = 1 + ∑ max [0,zi(P*)] to represent the denominator in equation (22).

Accordingly, $cP_i^* = P_i^* + Max\left[0, z_i\left(P^*\right)\right]$ 23

$$\left(c-1\right)P_i^* = Max\left[0, z_i\left(P^*\right)\right] \quad 24$$

But by definition, c ≥ 1

If $c = 1: P_i^* = Max\left[0, z_i\left(P^*\right)\right] = 0$ 25

Thus $P_i = 0 = Max\left[0, z_i\left(P^*\right)\right] \leq 0$ 26

and $z_i\left(P^*\right) \leq 0$

If $c > 1,: P_i^* > 0$. This implies that $z_i\left(P^*\right) > 0$. It follows in this case that

$$\sum_{i=1}^{N} P^* izi(P^*) > 0$$

Walras' Law is violated. But since c ≥ 1by definition, it follows that c must be equal to 1.

From equation (25) this implies that zi (P*) ≤ 0 ∀i = (1, ..., n) and this together with Walras'law means that all of the conditions for equilibrium hold at the fixed point P*. This completes our proof for pure exchange model. The proof of existence of market equilibrium becomes more complicated when production activities are included.

Moreso, the problem of existence of equilibrium can be demonstrated within a partial-equilibrium example of a demand and supply model. Equilibrium exists when at a certain positive price; the quantity demanded equates the quantity supplied. At such a price, there is neither excess demand nor excess supply.

5. Conclusion and Implications

This study has established that the general equilibrium and welfare economics run the spectrum from specific policy conclusions on particular issues like interdependence, interrelationship, rationing, institutional arrangement, to broad, rather than logical examination into the proper foundations for the whole area of investigation. General equilibrium theory despite its obvious shortcomings is the most complete existing model of economic behaviour. General equilibrium theory allows us to be aware of the tremendous complexity of the real world by viewing the economy as a vast system of mutually interdependent markets. Under certain assumptions, the general equilibrium model has a solution: it yields a set of price ratios, which leads to an optimal allocation of resources and welfare improvement. This solution and its optimality assumptions can be used as a norm determine the importance and implications of deviations of the various welfare criteria. For instance, the implication of the general equilibrium on is that, government should secure the competitive environment in an economy and give people full economic freedom so as to improve their welfare**.** This implication suggests that there should be no subsidy, no price floor, no price ceiling, stop a rent control, no regulations, and lift the tax and the import-export barriers.

One advantage of general equilibrium theory is that it focuses attention on the question of how to achieve economic efficiency that can improve the welfare of the people. The phenomenon also serves as a basis for simulations of the whole economy that give useful estimates of its evolution over time and of the impact of changes in taxes, technology, and resources and many more. Such simulations are examined in Computable General Equilibrium (CGE), and it has a huge literature. The study of general equilibrium theory is an in-road to understand major debates in economics, many of which are expressed in the language of the theory. Probably, the theory's most important use is to guide research by providing examples of conclusions that could be drawn or might have to be modified as the underlying assumptions of the model are made more realistic.

Consider the theorem that equilibria exist. First of all, it is crucial to understand that this theorem does not mean that actual economies are in equilibrium. The theorem cannot do so, because, like all theoretical statements, it is an assertion about a model. It would be equally illogical to argue on the contrary that equilibria exist in a general equilibrium model because actual markets clear. The model and economic reality are distinct entities. It is therefore sensible to check the realism of the theorem's assumptions so as to see whether the theorem well represents reality. This point of view is an indication that the theorem is inadequate to represent reality. This indicates the possibility of actual markets sometimes not clearing as a result of prolonged periods, namely recessions and depressions, when supply greatly exceeds demand in very important markets.

A trap to be avoided, in general, is to accept general equilibrium theory hock line and sinker. A healthier attitude is to think of the theory as tentative and to be modified as knowledge accumulates about how actual economies function. General equilibrium theory is a set of useful tautologies derived from microeconomics with a great deal of accumulating evidence that the basic assertions of the theory can be adjusted. The theory has, however, the advantage that it is simple and easy to use and remember.

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